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MATHEMATICAL MODELLING OF METAL DRAWING PROCESS

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Analysis of theoretic aspect of the influence of inertial lubricant forces on the process of cold metal drawing was performed mathematically. The influence of die structure properties, rheologic lubricant properties as well as kinematic influence of drawing speed and suggested lubricant removing device assembled in the die clearance was taken into consideration. Primary part of the analysis, was performed over pressure gradients in which analytical solutions of differential equations were incorporated.

Key words: *metal drawing process, mathematical model, lubricant, inertia*

Matematičko modeliranje izvlačenja metala. Izvedena je matematička analiza teorijskog pogleda utjecaja sila inercije podmazivanja na proces hladnog izvlačenja metala. U obzir je uzet utjecaj svojstava konstrukcije matrice, reoloških svojstava maziva, kinematički utjecaj brzine izvlačenja kao i predloženi uređaj za uklanjanje maziva nakupljenog u zazoru matrice. Temeljni dio analize odnosi se na gradijente tlaka u koje su uključena analitička rješenja diferencijalnih jednadžbi.

Ključne riječi: *izvlačenje metala, matematički model, mazivo, inercija*

INTRODUCTION

Contemporary trend of plastic processing of metal demands that operating speeds in technological process should be increased for the purpose of achieving a larger production [1]. There are reference analyses of inertial forces mainly based on theoretical investigations. For such an analysis performed by differential equations of O. Reynolds acceleration is to be added, so the analyses have notations as follows [2 - 4]:

$$\frac{\partial^2 v_x}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} + \frac{1}{\nu}\varpi \tag{1}$$

$$\frac{\partial p}{\partial x} = 0 \tag{2}$$

$$\frac{\partial p}{\partial x} = 6\mu\frac{v_0}{\varepsilon^2(x)} + C_1\frac{\mu}{\varepsilon^3(x)} + \mathrm{tg}\alpha\cdot\rho\frac{16v_0^2\varepsilon^2(x) - C_1^2}{120\varepsilon^3(x)} \tag{3}$$

$$C_1 = \frac{k}{2} - \sqrt{\frac{k^2}{4} + 2v_0\varepsilon_0(8v_0\varepsilon_0 + 3k)} \tag{4}$$

$$k = 120\frac{\nu}{\mathrm{tg}\alpha} \tag{5}$$

Functional dependence $\varepsilon(x)$ due to developing into MacLaurin's sequence may be expressed by notation [5]:

$$\varepsilon(x) = \varepsilon_0 - \alpha x + \frac{x^2}{2R_z} - \frac{\alpha x^3}{2R_z^2} + ... \tag{6}$$

The change of dynamic viscosity of pressure lubricant is expressed by Baruss formula:

$$\mu = \mu_0 e^{\gamma p_0} \tag{7}$$

Projection of speed vector of lubricant removing devices on the axis x is contained in the relation:

$$V_{G_x} = v_G\sqrt{1 - \left(\sin\alpha - \frac{x}{R}\right)^2} \cong v_G \tag{8}$$

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The length of lubricating wedge has approximately[5] the following notation in modeling [6]:

$$a \cong R_z \left[\sqrt{1-\left(\cos\alpha - \frac{\varepsilon_a-\varepsilon_0}{R_z}\right)^2} - \sin\alpha \right] \quad (9)$$

In the Figure1. a principal outline of metal drawing with lubrication is presented.

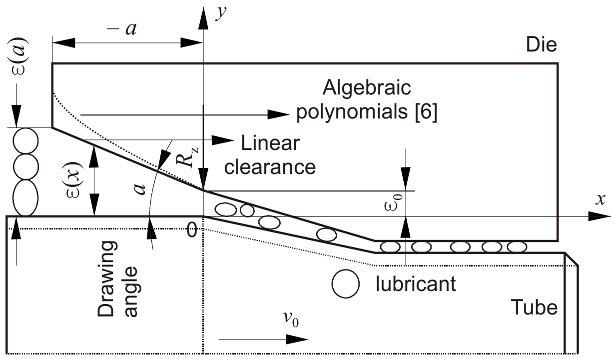

Figure 1. **Outline of metal drawing with lubrication**
Slika 1. **Shema izvlačenja metala s podmazivanjem**

Before the access to cross-section of deformation zone the clearance will be modeled by mathematical polynomials [7]. Mathematical analysis is directed to the Table 1. and the Figure 1.

Table 1. **Outline of analysis**
Tablica 1. **Shema analize**

Type of modeling	Variables
Modeling of construction properties of die	$R_z \longrightarrow$ radius of die modeling
	$a \longrightarrow$ length of lubricating wedge
	$\varepsilon_a \longrightarrow$ thickness of lubricant
	$x \longrightarrow$ active path of lubricant inertia
Kinematic	V_G and v_0
Rheologic properties of lubricant	μ, ρ and γ

POSITIONED MASTER GAUGES

Position of master gauges[1] for theoretical investigations is presented in Table 2.

Die structure plays a great role in processes of metal drawing [7].

[1] Different positioning of master gauges includes also different referential indices of inertia in relation to which the analysis is performed while special parameters of other master gauges can be brought by solver into the boundary value of master gauge 1.

Table 2. **Master gauges**
Tablica 2. **Glavna mjerila (kavezi)**

Nr. of master gauges	Technological properties	Solver or Goal Seek: Target Cells	Solver or Goal Seek: Changing Cells	Graphical or tabular notations
Master gauge 1	I = 1,0096215 $\nu = 2{,}52\cdot10^{-5}$ α=11° $\varepsilon_0^M = 1{,}311\cdot10^{-6}$ ρ = 954 $\varepsilon_a >> \varepsilon_0^M$ $p\cdot\gamma$ = 4,36 R_z = 0,01 μ = 0,0240408 $x = -5\cdot10^{-3}$ square clearance v_0 = 16 v_G = 0	I	μ x a	Figure 2. Equation (12) for structure properties
Master gauge 2	It follows from the first master gauge when effected upon by Goal Seek for I = 1,1	I	μ v_0 α ε_0^M	Figure 5.
Master gauge 3	The second master Gauge is effected upon by Solver	I	v_0 α x	Figure 3.
Master gauge 4	The third master gauge is effected upon by Solver $\varepsilon_a = 1{,}0\cdot10^{-5}$	I	ε_0 v_0 x R_z	Figure 4.
Master gauge 5	Cubic clearance is mounted on the fourth master gauge $v_G > v_0$	I	μ v_G v_0	Table 3.
Note - Subject to constraints: $\varepsilon_0 > 0, \varepsilon(x) < \varepsilon_a, R_z > \varepsilon_0^M, v_G > v_0$				

DISCUSSION OF RESULTS

Die clearance for linear and square approximation is calculated according to the master gauge 1 by formula (6). The results of analysis are presented in Figure 2. A stable hidrodynamic lubrication was regulated. Dynamic viscosity with the increase of inertia index decreases, while structure property increases.

In this connection the structure properties are pointed out more intensively , while both rheologic and structural properties are in general dominant in the zone of small inertia index as to increase and decrease in unique manner. Investigations of solver[2] are miscellaneous.

In Figure 3. a mutual relationships between structural and rheologic properties according to the master gauge 1 is presented and an interpolation chart is given. Relationships is rather complex, because there is an intensive fall

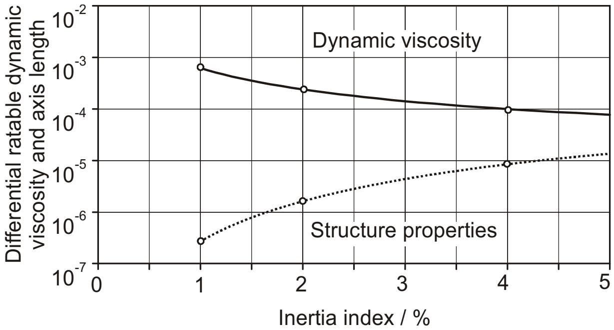

Figure 2. **Influence of rheologic and structural property on inertia index in comparison with linear and square clearance**
Slika 2. **Utjecaj reoloških i konstrukcijskih svojstava na indeks inercije u usporedbi s pravocrtnim i kvadratnim zazorom**

of lubricant rheologic properties on the short way on which the lubricant in accelerated which is pointed out by difference of linear and square clearance of lubricant before the access to deformation region. Expected exponential dependence of this chart did not answer satisfactorily. Investigations of solver are miscellaneous.

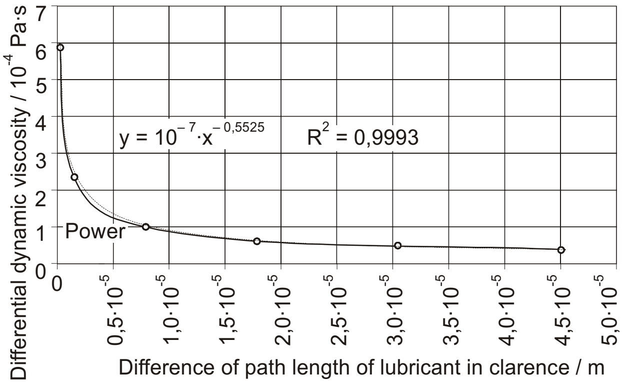

Figure 3. **Relationships between rheologic and structural properties at inertia in comparison with linear and square clearance**
Slika 3. **Odnos između reoloških i konstrukcijskih svojstava pri inerciji u usporedbi s pravocrtnirn i kvadratnim zazorom**

For greater inertia indices there is no significant difference in kinematic effect between square and cubic clearance. The thickness of a lubricant layer over which the evaluation of inertia index is given, is calculated by the formula from Mizuna-Grudeva and corrected according to the solution of differential equation through square clearance.

In Figure 4. the influence of the thickness of lubricant before the clearance on the inertia index at boundary lubricant is given. The investigations are interesting here so much more, because the thickness of lubricant layer (film) can be regulated so as to aim to monomolecular layer in order to get the thickness of lubricating film less than metal surface roughness. Although it is mathematically clearly defined that for this-kind analysis there is unambiguous flow, i.e. with decrease of the thickness of lubricant layer before the access to the deformation zone the thickness of lubricating film is decreased too at the access to cross section of deformation zone. In one-parameter analysis solver reveals small pulsations, which theoretically follow from the solutions of differential equations in multi-parametric analysis through simultaneous processing of rheologic, structure and kinematic parameters of technologic process.

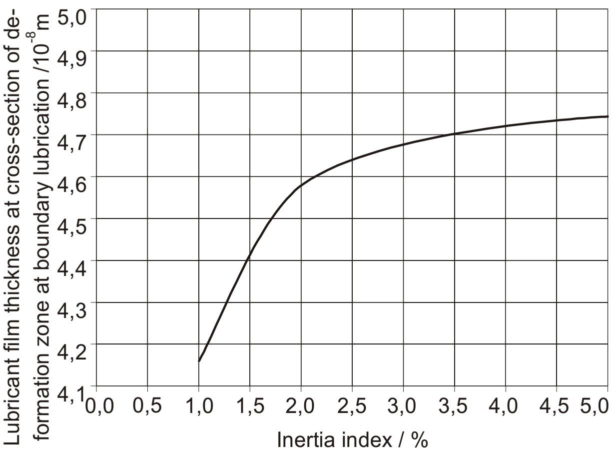

Figure 4. **Influence of lubricant thickness before the clearance on the lubricant thickness at the access to cross-section of deformation zone at boundary lubrication**
Slika 4. **Utjecaj debljine sloja maziva prije zazora na debljinu sloja maziva u graničnom podmazivanju u presjeku zone deformacije**

At this analysis, the index of inertia increases, the angle of drawing of material increases, radius of curvature of access to clearance increases unequally , and dynamic viscosity of lubricant decreases, until speed of metal drawing only changes in a pulsation manner. The speed of metal drawing may record a slight fall and preserve the increase of inertia. It is clear that here, in a specific synergic effect

Table 3. **Effects of lubricant removing device on master gauge 5**
Tablica 3. **Djelovanje uređaja za uklanjanje maziva na glavni mjerač 5**

Model of clearance	Speed of grab V_G / m·s^{-1} V_0 = 100 m·s^{-1}, R_G = 0,005 / m	Index of inertia
Linear clearance	126	1,05
Square clearance	143	1,05
Cubic clearance	148	1,05

[2] Under miscellaneous solver analysis is understood investigation of sub-master gauge and super-master gauge positioned inertia index of referent master gauge.

rheologic, structural and kinematic properties of technologic process are in action.

In Figure 5., a complex solution Q = 30,8 % of decreasing of rheologic (μ) properties of lubricant through

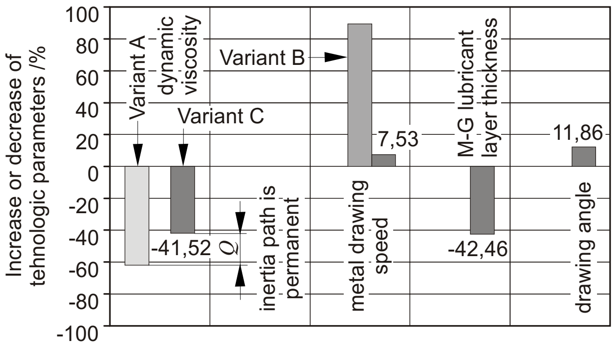

Figure 5. **Complex solution for master gauge 2 and theoretic increase of inertia index by 1 %**
Slika 5. **Složena otopina za glavni mjerač i teoretsko povećanje indeksa inercije za 1 %**

increasing of kinematic by 7,53 % (v_0), angle of drawing by 11,86 % and decreasing of lubricant thickness at the access to cross-section by 42,46 % is offered (ε_0). In this connection the index of inertia is increased by 1 %.

The results shown in Table 3. indicate that the assembly of a lubricant removing device would have the best effect in the linear clearance of die. Its structural solution is presented in the Figure 6.

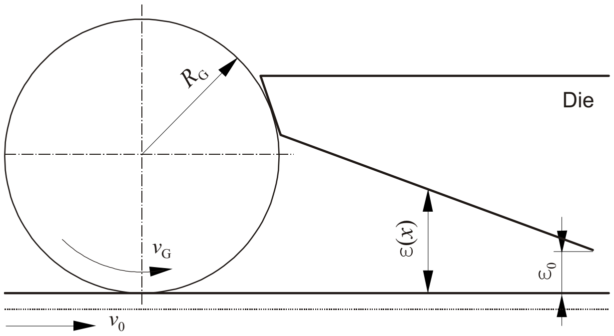

Figure 6. **Lubricant removing device**
Slika 6. **Uređaj za uklanjanje maziva**

Solution differential equation (3) for linear clearance is:

$$\frac{1-e^{-\gamma p_0}}{\mu_0\gamma} = \frac{6v_0}{\varepsilon_0 \mathrm{tg}\alpha} + \frac{1}{2\varepsilon_0^2 \mathrm{tg}\alpha}\left[\frac{k(k-1)}{2}(\Omega-1) - 6v_0\varepsilon_0 k\right] \quad (10)$$

$$\Omega = \mp\sqrt{\frac{k^2}{4} + 2v_0\varepsilon_0\left(8v_0\varepsilon_0 + 3k\right)} \quad (11)$$

Solution for square clearance first approximation is:

$$\frac{1-e^{-\gamma p_0}}{6\mu_0\gamma} = -\frac{\alpha}{2\varepsilon_0\left(\frac{2}{R}-\alpha^2\right)} + \frac{3\alpha}{2R\left(\frac{2}{R}\varepsilon_0-\alpha^2\right)^2} + \frac{1}{R\left(\frac{2}{R}-\alpha^2\right)\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}}\ln\frac{-\alpha-\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}}{-\alpha+\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}} - \frac{3\varepsilon_0}{2R^2\left(\frac{2}{R}\varepsilon_0-\alpha^2\right)^2\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}}\ln\frac{-\alpha-\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}}{-\alpha+\sqrt{\alpha^2-\frac{2}{R}\varepsilon_0}} \quad (12)$$

Chart showed at Figure 3. calculated at part abscise X where is maximal gradient pressure differential equation (3).

Chart showed at Figure 4. present solution differential equation (3) apply method Monte-Carlo.

Mathematical estimates for mineral oils and speeds drawing tubes up to 50 m/s show that we don't make bigger mistake from 1% in estimate ε_0 if we neglect influence inertia forces.

Technology drawing wire overcomes these speeds and therefore we should take analysis results in consideration.

CONCLUSIONS

Thanks to differently positioned master gauges[3] the process of cold metal drawing is analyzed. In this connection the properties of master gauges were altered so that the referent index of inertia of master gauges was inside the limits from 0 % through 5 % of inertial effect. In this way, sub-master gauges and super-master gauges analyses of Solver and Goal Seek were made on several operating examples so as to give a theoretic analysis of impact of rheologic, structural and kinematic effects on cold metal drawing with lubrication as convincing as possible.

Naturally, it was possible to make various parametric relationships by means of a series of diagrams because the analysis included eight alternating parameters. However, the presented situation is a good section of the complete analysis. A general from of regression could approximately express the relationship of rheologic properties of lubricants and structural properties of die [8].

$$\Delta Y = a\cdot\left(\Delta X\right)^{-b} \quad (13)$$

A synergetic impact of particular parameters on a basic parameter of analysis was detected, being well found

[3] Positioned master gauges are commonplace in theoretic investigations.

out by solver, which is illustrated in Figure 5. On the basis of the performed analysis, in a series of diagram notations no type of modeling is to be privileged according to Table 1., because they are complementary in the zones of small indices of inertia.

Modelling processes exercise a strong influence precisely at small inertia effects, which is nowadays taking place in the metallurgic manufacturing. At larger inertial effects, modeling of access to die clearance impacts less on inertial effect, domination being overtaken by kinetic effects. As presented in Table 3., the suggested assembly of lubricant removing device into the die clearance would have an effective influence on linear clearance.

Modelling process relying only on one type- if even through several parameters of analysis - is insufficient, because the whole analysis may be treated partially.

Alone access contains elements linear optimums approximately analytics solutions accordingly are adjusting Master gauges in Table 2.

On the base charts shows we can conclude that inertial forces are necessary in consideration for nowadays technologies operations.

List of symbols

x, y - coordinates of decartesian system / m,
p, p_0 - pressure / Pa,
μ, μ_0 - dynamic viscosity of lubricant / Pa·s,
ν - kinematic lubricant viscosity / m^2/s,
ρ - lubricant density / kg/m^3,
α - metal drawing angle / °,
$\varepsilon(x)$ - geometric characteristic / m,
ε_0 - lubricant layer thickness at access to deformation zone / m,
C_1 - the first rheologic property of lubricant / m^2/s,
k - the second rheologic property of lubricant / kg/ m^{2}s^2,
$\frac{\partial p}{\partial x}$ - gradient of pressure in lubricant film / kg/m^{2}s^2,
ε_a - lubricant layer thickness before deformation region / m,
γ - Piezo-coefficient of lubricant viscosity / Pa^{-1},
R_z - radius of modelling deformation region / m,
a - length of lubrication wedge / m,
$v_{0,} V_G$ - speed of metal drawing and speed of lubricant removing device before access cross-section of metal deformation region / m/s,
I - inertia index = $[\frac{\partial p}{\partial x}_{\text{equation (3)}}] / [\frac{\partial p}{\partial x}_{\text{equation (1)}}; \varpi = 0]$ / %,
ε_0^M - thickness of lubricant layer according to the Mizuna-Grudeva formula / m,
Q - complement of variant A and variant B in Figure 5. / Pa·s,
$M-G$ - corrected thickness of lubricant layer at access cross-section of zone of metal deformation / m
R^2 - regression coefficient,
ϖ - average acceleration / m/s^2.

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